On the Achievable Rate of Generalized Spatial Modulation Using Multiplexing Under a Gaussian Mixture Model

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***Abstract*—Spatial modulation (SM) is a new modulation technique where the information to be sent is encoded using one or more symbols and the subspace over which the symbols are transmitted. Unfortunately, a general capacity analysis that encompasses different forms of SM systems has not been developed. In this paper, we consider a general form of SM, where the number of transmitted data streams is allowed to vary. We refer to this form of SM as generalized spatial modulation with multiplexing (GSMM). A Gaussian mixture model (GMM) is shown to accurately model the transmitted spatially modulated signal using a precoding framework. Using this transmit model, a closed-form expression for the achievable rate when operating over Rayleigh fading channels is evaluated, and tight upper and lower bounds for the achievable rate are proposed. The expressions of the achievable rate obtained are flexible enough to accommodate any form of SM—where any subspace can be used for the transmission—by adjusting the precoding set. Simulations are presented to show the tightness of the proposed bounds. The effect of the system dimensions and a comparison with other prominent capacity results are also demonstrated in simulations.**

***Index Terms*—Achievable rate, spatial modulation, Gaussian mixture model, mutual information, precoding.**

I. Introduction

In the past 15 years, multiple-input multiple-output (MIMO) antenna systems have been studied extensively due to their ability to provide reliable increases in data rates compared to single antenna systems [1], [2]. With MIMO technology becoming mature, massive MIMO systems have been recently introduced and studied [3], [4]. Massive MIMO can increase the total throughput of the network by using a very large number of antennas in the transmitter or receiver.

Recently, spatial modulation (SM) has been proposed to enhance the spectral efficiency utilizing both digital modulation and MIMO technology [5]–[17]. In most SM work [5], [9], [10], [18], the transmitter activates only one of the transmit antennas per channel use. The scheme uses conventional amplitude/phase modulation (i.e., a conventional one-dimensional symbol is sent) and conveys additional information to the receiver embedded in the transmitting antenna index. In [19], a framework is introduced for the performance analysis of SM using order statistics.

There are two main advantages for one-dimensional SM over conventional MIMO spatial multiplexing in [11], [12]. These advantages are eliminating inter-channel interference (ICI) and removing the requirement of tight antenna synchronization as SM uses a single RF chain. Hence, the one-dimensional SM reduces both the transmission overhead and the receiver complexity compared to spatial multiplexing which improves energy efficiency [13] and makes it amenable to a large number of transmit antennas [14]. The receiver complexity reduction is emphasized in [5], [9], in which a sub-optimal SM detector is investigated. The sub-optimality of the detectors in [5], [9] causes an error floor, unless the fading channels are known at the transmitter as demonstrated in [10]. Sub-optimal SM detectors are summarized in [15], [16]. An optimal SM detector is proposed in [10]. Although optimal, the detector in [10] still has the same complexity order as other MIMO systems due to the joint detection of both spatial and modulated data. Therefore, low-complexity, optimal SM detectors have been developed in [15], [17] that separately treat detection of the antenna index and the modulation index.

Another approach that overcomes the problem of joint detection is a simplified version of SM known as space shift keying (SSK) [20]. In SSK, only the antenna index carries the information and needs to be detected but not the transmitted symbol. As a result, SSK dramatically reduces the detection complexity compared to SM, keeping comparable bit error rate performance, while the penalty is a reduction in spectral efficiency.

Instead of triggering one antenna for transmission in both SM and SSK, multiple antennas can be triggered using generalized spatial modulation (GSM) or generalized space shift keying (GSSK) as in [21] and [22], respectively. In GSM, the same symbol is transmitted on every antenna in an antenna subset and similarly in GSSK, multiple antennas can be triggered for transmission at the same time. When triggering more than one antenna at the same time for transmission, the interference among transmit antennas arises again, losing one of the main advantages of SM. However, ICI-free transmission is still preserved as mentioned in [6]. In [23]–[25], GSM was combined

Manuscript received August 13, 2015; revised November 17, 2015; accepted December 26, 2015. Date of publication January 7, 2016; date of current version April 13, 2016. The work of A. A. I. Ibrahim was supported by the Fulbright Scholarship. The work of T. Kim was supported by the Research Grant Council, Hong Kong, under Project CityU 21200714. The associate editor coordinating the review of this paper and approving it for publication was M. Matthaiou.

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Digital Object Identifier 10.1109/TCOMM.2016.2515624

with spatial multiplexing to transmit different data streams over the set of active transmit antennas leading to a hybrid of SM and spatial multiplexing. For this case, a near-optimal decoder is proposed in [24] where linear spatial multiplexing detectors are used.

In this paper, we present a general system model for spatial modulation, which we refer to as generalized spatial modulation with multiplexing (GSMM). In GSMM, the number of data streams transmitted is variable. GSMM utilizes precoding to generalize spatial modulation and encapsulate multistream spatial modulation using a fixed or variable number of data streams. The spatial information is assumed to be encoded on beamforming vectors (or matrices). Encoding spatial information on beamforming vectors was proposed earlier in [26] where a beamforming vector is chosen from a given codebook to convey spatial data, rather than the antenna index, however, the technique in [26] uses only a single data stream. The proposed precoding structure is more general and can handle a fixed or variable number of symbol streams. We show that the input vector of GSMM can be succinctly analyzed using a Gaussian mixture model.

The main contribution of this paper is the analysis of the capacity and the achievable rate for SM. Although the pairwise error probability has been widely investigated for SM, SSK, GSM, and GSSK in [9], [20]–[22], finding a general achievable rate expression is still an open problem. A tight upper bound for the capacity of SM systems was investigated in [27], in which only conventional SM with a single receive antenna was considered and was called information-guided channel hopping (IGCH). An upper bound is derived in [27] by separately treating the rate achieved by the antenna indices and the rate associated with the modulation size leading to a result that IGCH outperforms the single-input single-output (SISO) systems and orthogonal space-time coding (OSTBC) when the number of antennas is larger than two. The achievable rates of SM and SSK characterized through an empirical study were the main focus of the work in [28], demonstrating that SM is superior to a SISO system but inferior to a multiple-input single-output (MISO) system. GSM and GSSK, however, were not considered in [28]. Recently, a new capacity analysis was presented in [29], [30] where the spatial modulation system is modeled as two independent sources of information leading to a straightforward calculation of the system capacity. On the other hand, the analysis in this paper jointly models both sources of information using precoding and a Gaussian mixture model (GMM). There has been some work on the secrecy rate of SM systems in the presence of an eavesdropper in [31], [32]. Thus far, no work analyzing the capacity of a general SM framework encompassing SM, GSM, SSK, GSSK, and GSMM has been reported.

We present a general capacity analysis encompassing different forms of SM (conventional SM, GSM, SSK, GSSK and GSMM) along with tight upper and lower bounds of the achievable rate. The key idea that leads to the closed-form expression is to analytically treat the precoded GSMM data streams as a GMM random variable and employ the GMM distribution.

The rest of the paper is organized in the following way. In Section II, the system model is given. Section III discusses the GMM distribution, describing its probability density function (pdf) and providing a closed-form expression for its covariance matrix. In Section IV, the mutual information and entropy of the complex GMM random vector are studied with the derivation of an upper bound and a lower bound on the entropy. Achievable rate expressions are presented in Section V. Simulations exploring the obtained results are demonstrated in Section VI.

The following notations are admitted throughout this paper. Bold and lowercase letters denote vectors while bold and capital letters denote matrices. Lowercase letters that are not bold denote scalars. The notation $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose of a vector or a matrix, respectively. Also, $\det(\cdot)$ and $\text{trace}(\cdot)$ denote the determinant and the sum of the diagonal elements of a square matrix, respectively. The notation $\text{card}(\cdot)$ refers to the cardinality of a set while $|\cdot|$ is the absolute value of a scalar. By $\|\cdot\|_2$, we mean the Euclidean norm of a vector. The expectation of a random variable or vector is denoted by $E[\cdot]$.

II. System Model

Consider a MIMO system in Fig. 1, where the system is equipped with M_t transmit antennas and M_r receive antennas. The channel input-output expression is represented by

$$\boldsymbol{y} = \sqrt{\rho}\,\boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}, \tag{1}$$

where ρ is the signal-to-noise ratio (SNR), $\boldsymbol{y} \in \mathbb{C}^{M_r \times 1}$ is the received vector, $\boldsymbol{x} \in \mathbb{C}^{M_t \times 1}$ is the transmitted vector, $\boldsymbol{n} \in \mathbb{C}^{M_r \times 1}$ is the noise vector, and $\boldsymbol{H} \in \mathbb{C}^{M_r \times M_t}$ is the channel matrix. Let h_{ij} represent the flat-fading channel coefficient between the jth transmit antenna and the ith receive antenna. Perfect channel state information (CSI) is assumed at the receiver but not at the transmitter. The entries of $\boldsymbol{n}$ and $\boldsymbol{H}$ are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. The transmitted vector $\boldsymbol{x}$ is subject to the power constraint

$$\text{trace}\left(E\left[\boldsymbol{x}\boldsymbol{x}^H\right]\right) \leq 1. \tag{2}$$

When evaluating the capacity of conventional MIMO systems, the transmitted vector $\boldsymbol{x}$ is assumed to be a zero-mean complex Gaussian random vector. The Gaussian input assumption is chosen to maximize the mutual information between the transmitted and received vectors [1], [33], [34]. However, this is not the case when SM is used. In the following, we write the transmitted vector $\boldsymbol{x}$ in a form that enables the derivation of its pdf when either conventional or generalized SM is used.

This can be better depicted using a vector notation. Let the transmitted unit energy complex symbol be s where $E\left[|s|^2\right] = 1$. The transmitted vector $\boldsymbol{x}$ for conventional SM can be expressed as

$$\boldsymbol{x} = \boldsymbol{e}s,$$

where $\boldsymbol{e} \in \{\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_{M_t}\}$ with $\boldsymbol{e}_i$ denoting the ith column of the M_t-dimensional identity matrix. The choice of the vector $\boldsymbol{e}_i$ corresponds to transmission of the modulated symbol over antenna i while not transmitting over all other $M_t - 1$ antennas. The choice of i conveys information to the receiver.

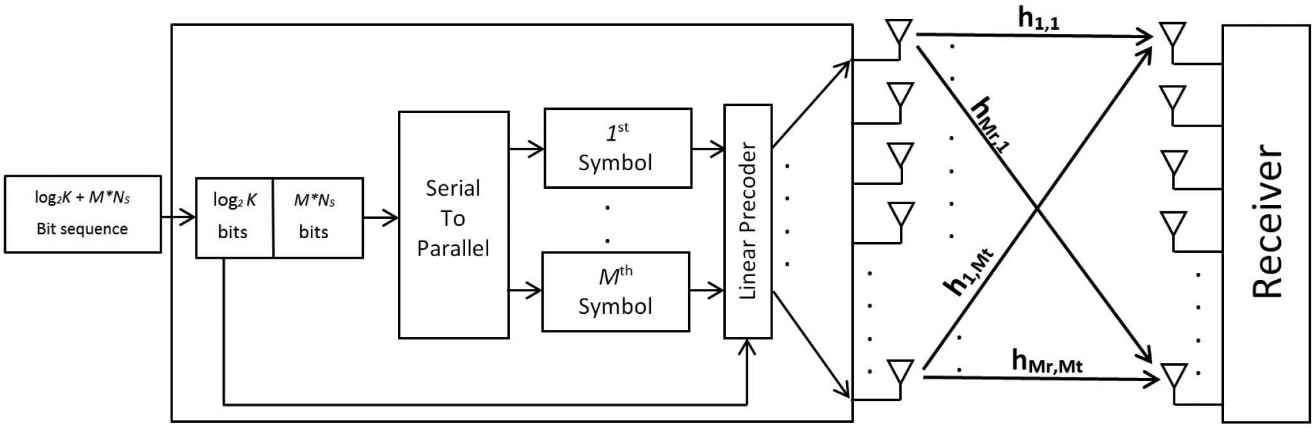

Fig. 1. System Model for GSMM where N_s is the number of bits per symbol.

In the case of the GSM in [21], more than one antenna can be triggered for transmission, leading to a transmitted vector of the form

$$\boldsymbol{x} = \boldsymbol{u}s, \tag{3}$$

where $\boldsymbol{u} \in \mathbb{C}^{M_t \times 1}$ is a unit-norm vector and $\boldsymbol{u} \in \mathscr{U} = \{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_{K_1}\}$. Here, K_1 is the cardinality of the set $\mathscr{U}$ that depends on the precoding structure. In the case of subset antenna selection, the vector $\boldsymbol{u}$ will have only N non-zero elements in the locations corresponding to the indices of the triggered antennas.

This can be easily extended to allow a form of generalized SM using spatial multiplexing where there are multiple input data streams. If the input data streams are $s_1, s_2, \ldots, s_M$ leading to an input symbol vector $\boldsymbol{s} = [s_1\ s_2 \ldots s_M]^T$ satisfying the unity power constraint. The transmitted vector $\boldsymbol{x}$ is then given by

$$\boldsymbol{x} = \boldsymbol{F}\boldsymbol{s}, \tag{4}$$

where $\boldsymbol{F} \in \mathbb{C}^{M_t \times M}$ such that $\boldsymbol{F} \in \mathscr{F}_M = \{\boldsymbol{F}_1, \boldsymbol{F}_2, \ldots, \boldsymbol{F}_{K_M}\}$ and similarly K_M depends on the precoding structure. The power constraint in (2) makes all the precoding matrices satisfy the condition

$$\text{trace}\left(E\left[\boldsymbol{F}_i \boldsymbol{s}\boldsymbol{s}^H \boldsymbol{F}_i^H\right]\right) \leq 1.$$

In the case of GSMM where a variable number of data streams is allowed, the precoding matrix $\boldsymbol{F}$ changes while the transmitted vector $\boldsymbol{x}$ is on the same form as in (4). The symbol vector $\boldsymbol{s}$ will be of variable dimension and equal to the number of input streams M (i.e., $\boldsymbol{s} \in \mathbb{C}^{M \times 1}$). The minimum dimension of the vector $\boldsymbol{s}$ is one (i.e., conventional SM where $M = 1$), and the maximum dimension does not exceed the total number of transmit antennas M_t (i.e., conventional spatial multiplexing). The precoding matrix $\boldsymbol{F}$ will have a variable size of $M_t \times M$ where the number of columns will be variable and $\boldsymbol{F} \in \mathscr{F}$. The set $\mathscr{F}$ is the set of all possible precoding matrices. In other words,

$$\mathscr{F} = \bigcup_{i=1}^{M_t} \mathscr{F}_i, \tag{5}$$

where $\mathscr{F}_i$ is the set of precoding matrices of dimension $M_t \times i$ corresponding to i input data streams while $\mathscr{F}_i$ is an empty set if i data streams are not to be transmitted. The cardinality of the set $\mathscr{F}$ is

$$K = \text{card}(\mathscr{F}) = \sum_{i=1}^{M_t} \text{card}(\mathscr{F}_i) = \sum_{i=1}^{M_t} K_i, \tag{6}$$

leading to a transmitted vector $\boldsymbol{x}$ given by

$$\boldsymbol{x} = \boldsymbol{F}'\boldsymbol{s}', \tag{7}$$

where $\boldsymbol{F}' \in \mathscr{F} = \{\boldsymbol{F}'_1, \boldsymbol{F}'_2, \ldots, \boldsymbol{F}'_K\}$ where K is the cardinality of the set $\mathscr{F}$ and $\boldsymbol{s}'$ is the data stream vector with variable dimension that is changing from 1 to M_t matching the number of columns of the precoding matrix $\boldsymbol{F}'$.

This case, which is described in (7), is the most general because it can be adjusted to describe any of the other SM scenarios. For example, if the number of data streams is fixed to M, then the union in (5) contains only the set $\mathscr{F}_M$. For the case of a single data stream ($M = 1$), the only non-empty set in the union is $\mathscr{F}_1$. Moreover, if only one antenna is triggered ($N = 1$), $\mathscr{F}_1$ will consist of the columns of the M_t-dimensional identity matrix. It is also worth mentioning that GSMM is more challenging at the receiver side not only because the number of data streams is not constant but also because the number of bits per stream varies as the number of data streams vary. The bits to be transmitted are split into two blocks, one block represents the modulated symbol (and this is fixed as long as we fix the modulation scheme) and the other one represents the spatial information (or antenna index) that depends on the cardinality of the precoding set that changes when we change the number of data streams as shown in (5) and (6).

It is straightforward to show that the precoding matrix set can be mapped to a set of covariance matrices $\mathscr{Q}$. For example, in the case of fixed number of data streams (M), the covariance matrix of the transmitted vector can be written as

$$\boldsymbol{Q}_i = E\left[\boldsymbol{x}\boldsymbol{x}^H \mid \boldsymbol{F} = \boldsymbol{F}_i\right] = E\left[\boldsymbol{F}_i \boldsymbol{s}\boldsymbol{s}^H \boldsymbol{F}_i^H\right] = \boldsymbol{F}_i \boldsymbol{F}_i^H, \tag{8}$$

for $i = 1, 2, \ldots, K_M$, where $\boldsymbol{Q}_i$ is the covariance of the transmitted vector $\boldsymbol{x}$ assuming that the symbol vector $\boldsymbol{s}$ has complex

Gaussian i.i.d. entries that have zero mean and unit variance. This means that the precoding matrix set has a one-to-one correspondence with the set of covariance matrices $\mathcal{Q}$. According to the transmitted information, the precoding matrix dimension and the set of possible precoding matrices will be determined which in turn determines the set of possible covariance matrices. In other words, we can consider spatial information to be conveyed in the covariance matrices instead of the precoding matrices (or vectors).

Utilizing this concept, the input vector $\boldsymbol{x}$ can be modeled using a GMM as shown in Section III. The GMM is a mixture of Gaussian distributions with each of the distributions having a certain probability to be chosen. The effect of the spatially modulated signals (encoded in the index of the precoding matrix) appears in the distribution of the transmitted vector that is distributed as a GMM. To be more explicit, if the input symbols to the channel have a Gaussian distribution, then the distribution of the transmitted vector will be a GMM if SM is used while it will be Gaussian if no spatially modulated data is transmitted. For example, in case of a MISO system with three transmit antennas using conventional SM where we trigger one antenna per transmission, we have three possible covariance matrices given by

$$\mathbf{Q}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{Q}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{Q}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where each of the three covariance matrices is chosen with a probability α_i, $i = 1, 2, 3$, respectively. If SM is not used and we are transmitting through one of the transmit antennas, then the covariance matrix of the transmitted signal will be only one of the three covariance matrices mentioned above (according to which antenna is chosen) with probability one. Being a mixture of Gaussian distributions makes a GMM a reasonable assumption when evaluating the achievable rate.

III. Gaussian Mixture Model (GMM) Distribution

In this section, the pdf of the transmitted vector $\boldsymbol{x}$ and covariance matrix are derived. The general case of SM mentioned in (4) is considered assuming a fixed number of data streams (M). This will be generalized to the case of GSMM with a variable number of data streams in Section V.

A. Probability Density Functions of the Spatially Modulated Transmitted and Received Vectors

The symbol vector $\boldsymbol{s}$ is assumed to be complex Gaussian with zero mean and identity covariance. This causes the transmitted vector $\boldsymbol{x}$ to be also complex Gaussian given a certain precoding matrix (i.e., $\boldsymbol{F} = \boldsymbol{F}_i$) in the form

$$\boldsymbol{x} = \boldsymbol{F}_i \boldsymbol{s}.$$

Thus, $\boldsymbol{x}$ has a conditional pdf $g_i(\boldsymbol{x})$ that is complex Gaussian with zero mean and a covariance matrix $\boldsymbol{Q}_i$ as shown in (8). This can be written, in the form of a conditional distribution, as

$$p(\boldsymbol{x} \mid \boldsymbol{F} = \boldsymbol{F}_i) = g_i(\boldsymbol{x}), \tag{9}$$

where

$$g_i(\boldsymbol{x}) = \frac{1}{\pi^{M_t} \det(\boldsymbol{Q}_i)} \exp\left(-\boldsymbol{x}^H \boldsymbol{Q}_i^{-1} \boldsymbol{x}\right). \tag{10}$$

Due to the fact that the covariance matrices ($\boldsymbol{Q}_i$'s) have different ranks and might be singular in some cases (for instance, in the case of conventional SM, the rank of each of the $\boldsymbol{Q}_i$'s is one), the transmitted vector $\boldsymbol{x}$ can alternatively be described using its moment generating function $\Phi_i(\boldsymbol{\lambda})$ as

$$\Phi_i(\boldsymbol{\lambda}) = \exp\left(-\frac{1}{4}\boldsymbol{\lambda}^H \boldsymbol{Q}_i \boldsymbol{\lambda}\right).$$

On the other hand, the choice of a certain precoding matrix $\boldsymbol{F}_i$ conveys information and is also random. The probability mass function (pmf) of the random vector $\boldsymbol{F}$ can be assumed to be

$$p(\boldsymbol{F} = \boldsymbol{F}_i) = \alpha_i, \; i = 1, 2, \ldots, K_M, \tag{11}$$

where $0 \le \alpha_i \le 1$ and $\sum_{i=1}^{K_M} \alpha_i = 1$. Using (9), (10), and (11), we can write the following theorem that gives the pdf of the transmitted vector.

Theorem 1: If the transmitted symbol vector $\boldsymbol{s}$ has complex Gaussian entries with zero mean and unit variance, the transmitted vector $\boldsymbol{x}$ in the GSM system follows a complex GMM distrbution with pdf

$$g(\boldsymbol{x}) = \sum_{i=1}^{K_M} \alpha_i \, g_i(\boldsymbol{x}),$$

where $g_i(\boldsymbol{x})$ and α_i are as defined in (9) and (11), respectively.

Proof: Using (9), the cumulative distribution function (cdf) of the transmitted vector $\boldsymbol{x}$ conditioned on the precoding matrix $\boldsymbol{F}$ can be found to be

$$P(\boldsymbol{x} \le \boldsymbol{t} \mid \boldsymbol{F} = \boldsymbol{F}_i) = \int_{-\infty}^{t_{M_t}} \cdots \int_{-\infty}^{t_1} g_i(\boldsymbol{x}) \, dx_1 \cdots dx_{M_t},$$

where $i = 1, 2, \ldots, K_M$ and the notation $\boldsymbol{x} \le \boldsymbol{t}$ denotes that $x_1 \le t_1, x_2 \le t_2, \cdots, x_{M_t} \le t_{M_t}$. Using (11),

$$P(\boldsymbol{x} \le \boldsymbol{t}, \boldsymbol{F} = \boldsymbol{F}_i) = \alpha_i \int_{-\infty}^{t_{M_t}} \cdots \int_{-\infty}^{t_1} g_i(\boldsymbol{x}) \, dx_1 \cdots dx_{M_t}, \tag{12}$$

where $i = 1, 2, \ldots, K_M$. This will lead to the marginal cdf, $G(\boldsymbol{t}) = p(\boldsymbol{x} \le \boldsymbol{t})$, given by

$$G(\boldsymbol{t}) = \sum_{i=1}^{K_M} \left(\alpha_i \int_{-\infty}^{t_{M_t}} \cdots \int_{-\infty}^{t_1} g_i(\boldsymbol{x}) \, dx_1 \cdots dx_{M_t}\right).$$

Hence, the pdf of the transmitted vector $\boldsymbol{x}$ can be obtained from the cdf using the fundamental theorem of calculus to be

$$g(\boldsymbol{x}) = \sum_{i=1}^{K_M} \alpha_i \, g_i(\boldsymbol{x}). \tag{13}$$

The expression in (13) matches the pdf of a complex GMM random vector with zero means and a covariance matrix set $\mathcal{Q} = \{\boldsymbol{Q}_1, \boldsymbol{Q}_2, \ldots, \boldsymbol{Q}_{K_M}\}$. ■

This theorem introduces a very important result that is the key of our analysis in this paper. It shows that the unconstrained assumption (zero-mean Gaussian assumption) of the symbol vector fed into the precoder leads a transmitted vector $\boldsymbol{x}$ that is a GMM random vector when spatial modulation is employed (in the form of information carrying precoders). Using Theorem 1, it is straight forward to show that the received vector $\boldsymbol{y}$ is also distributed as a complex GMM distribution with pdf

$$f(\boldsymbol{y}) = \sum_{i=1}^{K_M} \alpha_i \ f_i(\boldsymbol{y}).$$

This is due to the fact that the received vector $\boldsymbol{y}$, given a certain precoding matrix $\boldsymbol{F}_i$ is used for transmission, is of the form

$$\boldsymbol{y} = \sqrt{\rho}\boldsymbol{H}\boldsymbol{F}_i\boldsymbol{s} + \boldsymbol{n}.$$

Hence, $\boldsymbol{y}$ is conditionally distributed as a complex Gaussian random vector of dimension M_r with zero mean and a covariance matrix $\boldsymbol{\Sigma}_i$ giving

$$p(\boldsymbol{y} \mid \boldsymbol{F} = \boldsymbol{F}_i) = f_i(\boldsymbol{y}) = \frac{1}{\pi^{M_r} \det(\boldsymbol{\Sigma}_i)} \exp\left(-\boldsymbol{y}^H \boldsymbol{\Sigma}_i^{-1} \boldsymbol{y}\right), \tag{14}$$

where

$$\Sigma_i = \rho \boldsymbol{H}\boldsymbol{Q}_i\boldsymbol{H}^H + \boldsymbol{I}_{M_r}, \tag{15}$$

and $\boldsymbol{I}_{M_r}$ is the M_r-dimensional identity matrix. The marginal pdf of the received vector $\boldsymbol{y}$ can be found to be

$$f(\boldsymbol{y}) = \sum_{i=1}^{K_M} \alpha_i \ f_i(\boldsymbol{y}), \tag{16}$$

which is of the same form as that of a complex GMM random vector of dimension M_r with K_M complex Gaussian components of zero means and a covariance matrix set $\mathscr{E} = \{\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2, \ldots, \boldsymbol{\Sigma}_{K_M}\}$.

The investigation of the achievable rate involves the computation of mutual information. Mutual information is a function of the covariance matrix of the received vector. The covariance matrices of the transmitted and the received random vectors that are distributed as complex GMM are of interest.

B. Covariance Matrix of a GMM Random Vector

The next lemma provides a closed form expression of the covariance matrix.

Lemma 1: The covariance matrix $\boldsymbol{Q}$ of the complex GMM random vector $\boldsymbol{x}$ with pdf $g(\boldsymbol{x})$ as defined in (13) is

$$\boldsymbol{Q} = \sum_{i=1}^{K_M} \alpha_i \boldsymbol{Q}_i. \tag{17}$$

Proof: As the transmitted vector $\boldsymbol{x}$ has zero mean, its covariance matrix will be the same as its autocorrelation matrix. For $\boldsymbol{x}$, the covariance matrix $\boldsymbol{Q}$ will be

$$\boldsymbol{Q} = E\left[\boldsymbol{x}\boldsymbol{x}^H\right].$$

This can be evaluated using the expectation conditioned on the precoding matrix $\boldsymbol{F}$ described as

$$E\left[\boldsymbol{x}\boldsymbol{x}^H\right] = E_{\boldsymbol{F}}\left[E\left[\boldsymbol{x}\boldsymbol{x}^H \mid \boldsymbol{F}\right]\right].$$

However, the conditional expectation can be found to be $E\left[\boldsymbol{x}\boldsymbol{x}^H | \boldsymbol{F} = \boldsymbol{F}_i\right] = \boldsymbol{Q}_i$, which leads to

$$\boldsymbol{Q} = E\left[\boldsymbol{x}\boldsymbol{x}^H\right] = \sum_{i=1}^{K_M} \alpha_i \boldsymbol{Q}_i. \tag{18}$$

This concludes the proof. ■

Lemma 1 is general and can be extended to apply to the received vector $\boldsymbol{y}$ because it is also a complex GMM random vector. Replacing $\boldsymbol{Q}_i$ in (18) by $\boldsymbol{\Sigma}_i$ leads to the covariance matrix of the received vector $\boldsymbol{y}$

$$\boldsymbol{\Sigma} = E\left[\boldsymbol{y}\boldsymbol{y}^H\right] = \sum_{i=1}^{K_M} \alpha_i \boldsymbol{\Sigma}_i = \sum_{i=1}^{K_M} \alpha_i \left(\rho \boldsymbol{H}\boldsymbol{Q}_i\boldsymbol{H}^H + \boldsymbol{I}_{M_r}\right).$$

These results will be useful when analyzing the mutual information between the transmitted vector $\boldsymbol{x}$ and the received vector $\boldsymbol{y}$ of GSMM.

IV. Mutual Information of GMM Random Variable

The informed mutual information between the transmitted vector $\boldsymbol{x}$ and the received vector $\boldsymbol{y}$ is written as

$$I(\boldsymbol{x}; \boldsymbol{y} \mid \boldsymbol{H}) = H(\boldsymbol{y} \mid \boldsymbol{H}) - H(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{H}), \tag{19}$$

where $H(\cdot)$ indicates the entropy function.

The differential entropy of $\boldsymbol{y}$ given $\boldsymbol{H}$ is given in [35] by

$$H(\boldsymbol{y} \mid \boldsymbol{H}) = E\left[-\log f(\boldsymbol{y})\right] = -\int_{\mathbb{C}^{M_r}} f(\boldsymbol{y}) \log f(\boldsymbol{y}) \, d\boldsymbol{y}. \tag{20}$$

From (20) and the definition of the pdf of a GMM random vector in (13), it is easy to see that there is no closed-form for the entropy of a vector with a GMM pdf. This is due to the logarithm of the sum of exponentials which can't be simplified [36].

Using the multivariate Taylor-series expansion of the logarithm of the sum as suggested in [36] assuming that all mixture components have zero mean gives

$$\log f(\boldsymbol{y}) = \sum_{k=0}^{L} \frac{1}{k!} \left(\boldsymbol{y}^T \ \nabla\right)^k \log f(\boldsymbol{y})|_{\boldsymbol{y}=\boldsymbol{0}} + O_L, \tag{21}$$

where L is the number of terms to be considered from the expansion, ∇ is the gradient with respect to the random variable $\boldsymbol{y}$, O_L is the remainder term, and the substitution by $\boldsymbol{y} = \boldsymbol{0}$ is only done inside the logarithmic function in the right hand side. Truncating the remainder term O_L in (21) yields the required finite approximation of the logarithm.

It is difficult to derive the deviation of the approximated entropy from its true value. Tight lower and upper bounds on entropy was investigated in [36]. The bound characterization makes it easy to tell if the obtained approximation is meaningful or not. An upper bound for the differential entropy of a complex GMM random vector is found by following the same steps that are used in [36] to prove the entropy upper bound for a real GMM random vector. An upper bound of the differential entropy of the received complex GMM random vector $\boldsymbol{y}$ is

$$H(\boldsymbol{y} \mid \boldsymbol{H}) \leq \sum_{i=1}^{K_M} \alpha_i \left(-\log \alpha_i + \log \left[(\pi e)^{M_r} \det (\boldsymbol{\Sigma}_i) \right] \right) = H_u(\boldsymbol{y} \mid \boldsymbol{H}). \tag{22}$$

A lower bound of $H(\boldsymbol{y} \mid \boldsymbol{H})$ can be obtained using the lower bound found in [36] (see Theorem 2 in [36]). This yields

$$H(\boldsymbol{y} \mid \boldsymbol{H}) \geq -\sum_{i=1}^{K_M} \alpha_i \log \left(\sum_{j=1}^{K_M} \alpha_j \gamma_{i,j} \right) = H_L(\boldsymbol{y} \mid \boldsymbol{H}), \tag{23}$$

where $\gamma_{i,j} = 1/\left(\pi^{M_r} \det\left(\boldsymbol{\Sigma}_i + \boldsymbol{\Sigma}_j\right)\right)$.

The upper bound obtained in (22) is not tight. We use the algorithm mentioned in [36] to refine the obtained upper bound. The algorithm successively merges Gaussian components of the GMM to identify Gaussian-shaped clusters. The algorithm then calculates the upper bound and compares with the current lowest upper bound. At each step, the algorithm merges two Gaussian components of the GMM.

Having the differential entropy (and hence, the mutual information) approximated and bounded, we can move to finding a closed-form along with bounds of the achievable rate of GSMM in the next section.

V. Achievable Rate Analysis

In this section, achievable rate analysis of the MIMO system employing GSMM that is described in the normalized model in (1) is provided. The term achievable rate here will be used for the mutual information between the transmitted and received vectors under the assumption that the transmitted vector has a complex GMM distribution.

A. Achievable Rate of GSM With a Fixed Number of Data Streams

The mutual information in (19) has two terms. The first term is the differential entropy $H(\boldsymbol{y} \mid \boldsymbol{H})$, and it can be found using the entropy results in (20) and (21) and bounded using (22) and (23) because the received signal vector $\boldsymbol{y}$ is a complex GMM random vector.

On the other hand, the second term is $H(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{H})$ which can be found, using the fact that the system is normalized and that the complex Gaussian noise vector has unit covariance matrix, to be

$$H(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{H}) = H(\boldsymbol{n}) = M_r \log(\pi e). \tag{24}$$

It is clear then that the first term is the only part in the mutual information expression in (19) that depends on the GMM assumption of the transmitted vector. Knowing the pdf of the received signal vector $\boldsymbol{y}$ that is mentioned in (16), the differential entropy $H(\boldsymbol{y} \mid \boldsymbol{H})$ can be written as

$$\begin{aligned} H(\boldsymbol{y} \mid \boldsymbol{H}) &= -\int_{\mathbb{C}^{M_r}} \log f(\boldsymbol{y}) \sum_{i=1}^{K_M} \alpha_i f_i(\boldsymbol{y}) \, d\boldsymbol{y} \\ &\approx -\int_{\mathbb{C}^{M_r}} \sum_{k=0}^{L} \frac{1}{k!} \left(\boldsymbol{y}^T \nabla \right)^k \log f(\boldsymbol{y}) |_{\boldsymbol{y}=\boldsymbol{0}} \\ &\quad \times \sum_{i=1}^{K_M} \alpha_i f_i(\boldsymbol{y}) \, d\boldsymbol{y}, \end{aligned} \tag{25}$$

where the log function is approximated using (21) by truncating the remainder term. This leads to a closed-form of the achievable rate obtained by subtracting (24) from (25) to give

$$C^M = H(\boldsymbol{y} \mid \boldsymbol{H}) - M_r \log(\pi e),$$

where C^M denotes the achievable rate of the GSM system assuming a fixed number of data streams equal to M multiplexed over the triggered transmit antennas ($N = M$). This approximated expression needs many terms of the Taylor series expansion which makes it impractical to use. To overcome the complexity of finding the approximated expression, a tight upper and lower bounds are investigated.

A tight upper bound for $H(\boldsymbol{y} \mid \boldsymbol{H})$ can be found using (22). An upper bound for the achievable rate assuming a fixed number of data streams (M) (denoted by C_u^M) can be shown using (19), (22), and (24) to be

$$\begin{aligned} C_u^M &= \sum_{i=1}^{K_M} \alpha_i \left(-\log \alpha_i + \log \left[(\pi e)^{M_r} \det (\boldsymbol{\Sigma}_i) \right] \right) \\ &\quad - M_r \log(\pi e). \end{aligned} \tag{26}$$

The summation can be split into two added terms where the logarithm can be distributed as follows

$$\begin{aligned} C_u^M &= -\sum_{i=1}^{K_M} \alpha_i \log \alpha_i + M_r \sum_{i=1}^{K_M} (\alpha_i \log(\pi e)) \\ &\quad + \sum_{i=1}^{K_M} (\alpha_i \log \det (\boldsymbol{\Sigma}_i)) - M_r \log(\pi e). \end{aligned} \tag{27}$$

Using (15) along with the fact that $\sum_{i=1}^{K_M} \alpha_i = 1$, the upper bound of the achievable rate becomes

$$C_u^M = \sum_{i=1}^{K_M} \left(\alpha_i \log \frac{1}{\alpha_i} + \alpha_i \log \det \left(\rho \boldsymbol{H} \boldsymbol{Q}_i \boldsymbol{H}^H + \boldsymbol{I}_{M_r} \right) \right). \tag{28}$$

This upper bound can be refined using the algorithm mentioned in Section IV.

On another front, a lower bound on the achievable rate, denoted by C_L^M, can be obtained using (23) to be

$$C_L^M = -\sum_{i=1}^{K_M} \alpha_i \log \left(\sum_{j=1}^{K_M} \alpha_j \gamma_{i,j} \right) - M_r \log(\pi e). \tag{29}$$

In general, the mixing variables $\{\alpha_i\}_{i=1}^{K_M}$ will be equal with $\alpha_i = \frac{1}{K_M}$. This will simplify the obtained upper bound of the achievable rate to

$$C_u^M = \log(K_M) + \sum_{i=1}^{K_M} \frac{1}{K_M} \log\det\left(\rho \boldsymbol{H}\boldsymbol{Q}_i\boldsymbol{H}^H + \boldsymbol{I}_{M_r}\right), \quad (30)$$

and the lower bound of the achievable rate will simplify to

$$C_L^M = -\frac{1}{K_M}\log\left(\prod_{i=1}^{K_M}\left(\sum_{j=1}^{K_M}\frac{1}{\pi^{M_r}\det\left(\boldsymbol{\Sigma}_i+\boldsymbol{\Sigma}_j\right)}\right)\right) + \log K_M - M_r\log(\pi e), \quad (31)$$

where $\boldsymbol{\Sigma}_i, i = 1, 2, \cdots, K_M$ can be found from the expression in (15). One remark to be made is that quantifying the rate sources (i.e., the rate achieved by the spatial modulation and the rate achieved by the conventional modulation) would be helpful towards optimizing the spatial modulation system. The separation can be seen by looking at the upper bound expression in (30) where the first term ($\log K_M$) models the achievable rate due to SM while the second term represents the rate due to the conventional symbols. However, it might not be useful to do the separation in this paper as the approach that is followed jointly represents the conventionally modulated data with the spatially encoded data through the GMM distribution.

B. Achievable Rate of GSMM

In GSMM, the number of data streams is assumed to be variable. This means that M is not fixed but it varies to take values $M = 1, 2, \cdots, M_t$. The choice of the number of data streams is assumed to be random with a uniform distribution. Hence, we can define the pmf of the discrete random variable M to be

$$p(M = j) = \frac{1}{M_t}, \ j = 1, 2, \cdots, M_t.$$

For each number of data streams (a realization of M where $M = i$), there exists a corresponding coding set $\mathscr{F}_i$ that has K_i precoding matrices. This leads to K_M being a random variable that has pmf

$$p(K_M = K_j) = \frac{1}{M_t}, \ j = 1, 2, \cdots, M_t. \quad (32)$$

Using the pmf in (32), the achievable rate of GSMM, denoted by C, can be written as

$$C = \sum_{j=1}^{M_t} C^j p(M = j) = \frac{1}{M_t}\sum_{j=1}^{M_t} C^{\{M=j\}}.$$

Similarly, the upper and lower bounds become

$$C_u = \frac{1}{M_t}\sum_{j=1}^{M_t} C_u^j,$$

and

$$C_L = \frac{1}{M_t}\sum_{j=1}^{M_t} C_L^j,$$

respectively, where $\boldsymbol{Q}_i \in \mathscr{Q}_j$ with $\mathscr{Q}_j$ denoting the subset of possible covariances given that j data streams are transmitted and $\{\boldsymbol{Q}_i\}_{i=1}^{K_j}$ are the elements of the set $\mathscr{Q}_j$.

C. Uniform Triggering of Transmitting Array

In (30), the construction of the precoding matrix set is implicit. If the precoding is assumed to uniformly trigger the antennas, motivated by unknown CSI at the transmitter, this will constrain the cardinality of the set of possible precoding matrices and limit the possible covariance matrices as well. In this subsection, we analyze GSMM under the assumption of uniform antenna triggering.

To be more specific about what is meant by uniform antenna triggering, we give this example. If $M_t = 3$, for instance, the uniform triggering leads to a set of possible covariance matrices $\mathscr{Q}$ given by

$$\mathscr{Q} = \bigcup_{i=1}^{3}\mathscr{Q}_i,$$

where

$$\mathscr{Q}_1 = \left\{\boldsymbol{Q}_{1,1}, \boldsymbol{Q}_{2,2}, \boldsymbol{Q}_{3,3}\right\},$$

$$\mathscr{Q}_2 = \left\{\frac{1}{2}\boldsymbol{Q}_{1,1}+\frac{1}{2}\boldsymbol{Q}_{2,2}, \frac{1}{2}\boldsymbol{Q}_{1,1}+\frac{1}{2}\boldsymbol{Q}_{3,3}, \frac{1}{2}\boldsymbol{Q}_{2,2}+\frac{1}{2}\boldsymbol{Q}_{3,3}\right\}, \text{ and}$$

$$\mathscr{Q}_3 = \left\{\frac{1}{3}\boldsymbol{Q}_{1,1}+\frac{1}{3}\boldsymbol{Q}_{2,2}+\frac{1}{3}\boldsymbol{Q}_{3,3}\right\}$$

with $\boldsymbol{Q}_{i,j} \in \mathbb{C}^{M_t \times M_t}$ denoting a sparse matrix where all the elements are zero except for the element in the ith row and jth column which is equal to one.

It is clear (as shown above in the example of $M_t = 3$) that the number of possible ways to transmit M data streams over M uniformly triggered antennas is

$$K_M = \binom{M_t}{M}, \quad (33)$$

leading to a total number of possible ways for transmission equal to

$$K = \sum_{i=1}^{M_t} K_i = \sum_{i=1}^{M_t}\binom{M_t}{i} = 2^{M_t} - 1. \quad (34)$$

The new upper bound C_u for a certain realization of M where $M = \nu$ can be rewritten as follows

$$C_u^\nu = \log(K_\nu) + \sum_{i=1}^{K_\nu}\frac{1}{K_\nu}\log\det\left(\rho\boldsymbol{H}\boldsymbol{Q}_{\nu,i}\boldsymbol{H}^H + \boldsymbol{I}_{M_r}\right),$$

where $\boldsymbol{Q}_{\nu,i}$ is the ith element in the set $\mathscr{Q}_\nu$. On the other hand, the lower bound is

$$C_L^\nu = -\frac{1}{K_\nu}\log\left(\prod_{i=1}^{K_\nu}\left(\sum_{j=1}^{K_\nu}\frac{1}{\pi^{M_r}\det\left(\boldsymbol{\Sigma}_{\nu,i}+\boldsymbol{\Sigma}_{\nu,j}\right)}\right)\right) + \log K_\nu - M_r\log(\pi e), \quad (35)$$

where $\boldsymbol{\Sigma}_{\nu,i} = \rho\boldsymbol{H}\boldsymbol{Q}_{\nu,i}\boldsymbol{H}^H + \boldsymbol{I}_{M_r}, i = 1, 2, \cdots, K_\nu$.

To be able to give an expression for the upper bound, the achievable rate upper bound is averaged over all possible realizations of M which gives

$$C_u = \sum_{\nu=1}^{M_t} C_u^{\nu} p\left(M=\nu\right),$$

yielding

$$C_u = \frac{1}{M_t}\sum_{\nu=1}^{M_t} C_u^{\nu}$$

and similarly,

$$C_L = \frac{1}{M_t}\sum_{\nu=1}^{M_t} C_L^{\nu}.$$

Furthermore, the achievable rate can be optimized in case of fixing the number of data streams by choosing the number of data streams that maximizes the lower bound. This can be done by choosing M_{opt} to be

$$M_{opt} = \underset{\nu\in\{1,2,\ldots,M_t\}}{\text{argmax}} \quad C_L^{\nu}.$$

VI. Simulations

In this section, we provide a set of simulation results to support the analysis presented in the previous sections. The scenario of uniform antenna triggering is assumed to give more insightful comparisons although the achievable rate approximation and bounds proposed can be used with any precoding structure. The tightness of the upper and lower bounds obtained is demonstrated in Fig. 2–4. In Fig. 2, the upper and lower bounds of the achievable rates of conventional SM (where only one antenna is triggered per transmission, i.e., $N = 1$, and one data stream is transmitted, i.e., $M = 1$) are shown for MISO systems with different dimensions. The same is done for GSM with spatial multiplexing over the activated antennas with $N = M = 2$, and for GSMM in Fig. 3 and Fig. 4, respectively.

It is clear from these figures that the bounds are tighter when the dimensions of the system (number of antennas) is smaller. There is approximately one bit/sec/Hz difference between the upper and lower bounds when the number of transmit antennas is four or less. This difference increases as the dimensions of the system increases. Another observation to be made is that the lower bound is almost the same for different system dimensions which contradicts the expected result that the achievable rate should increase when the dimensions of the system increases because of the transmission of spatial data along with the regularly modulated data. This shows that the lower bound loosens as the dimensions increase while the upper bound would still be tight due to using the refinement algorithm mentioned in Section IV.

In Fig. 5, the effect of the number of antennas to be triggered per transmission in a MISO system with 8 available transmit antennas utilizing GSM with a single data stream is demonstrated. Fig. 6 shows the same effect when GSM with multiple data streams that are spatially multiplexed over the triggered antennas (i.e., $N = M$) is used.

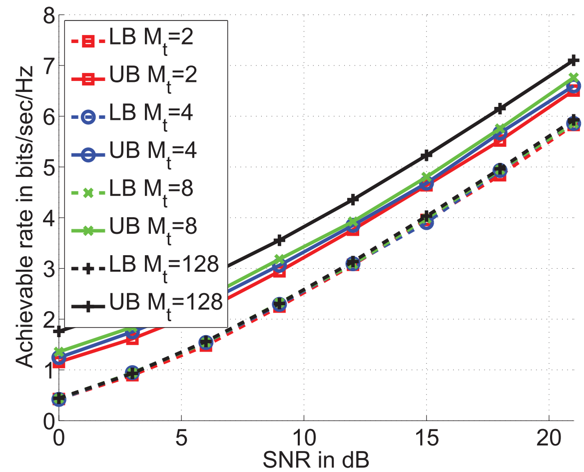

Fig. 2. Upper and lower bounds of the achievable rate of the conventional SM with a single data stream and one activated transmit antenna ($N = M = 1$) in MISO systems with $M_t = 2, 4, 8$, and 128 where LB and UB stand for lower bound and upper bound, respectively.

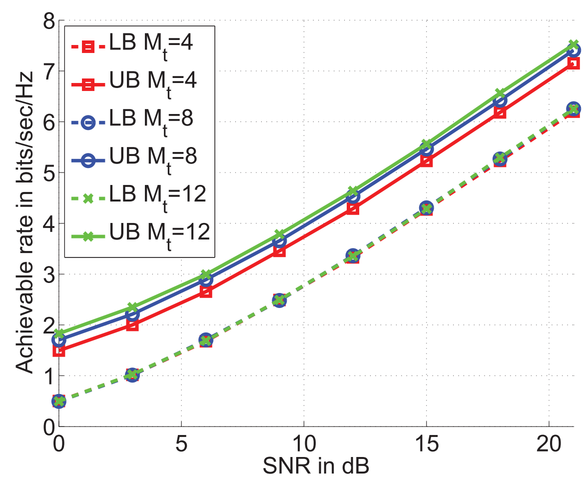

Fig. 3. Upper and lower bounds of the achievable rate of GSM with two transmit antennas activated at a time and two data streams ($N = M = 2$) in MISO systems with $M_t = 4, 8$, and 12.

GSM with a single data stream in Fig. 5 shows that triggering two antennas per transmission out of the eight available transmit antennas gives the same performance as triggering six out of the eight available. This is expected due to the fact that we are transmitting only a single stream and the cardinality of the precoding set in both cases (triggering two antennas or six antennas per transmission) is the same as $\binom{8}{2} = \binom{8}{6} = 28$. The achievable rate (we mean the upper bound as we mentioned earlier that the lower bound is less sensitive to the changes in the system) increases slightly when the number of triggered antennas per transmission is four out of the eight available transmit antennas. This demonstrates that triggering more than half of the number of available transmit antennas per transmission in the case of GSM with a single data stream is not helping the achievable rate while the energy efficiency is negatively affected. In SM, the energy efficiency can be defined, for instance in [37], as the achievable throughput over the total power consumed (including circuitry power consumption). Hence, turning on more RF

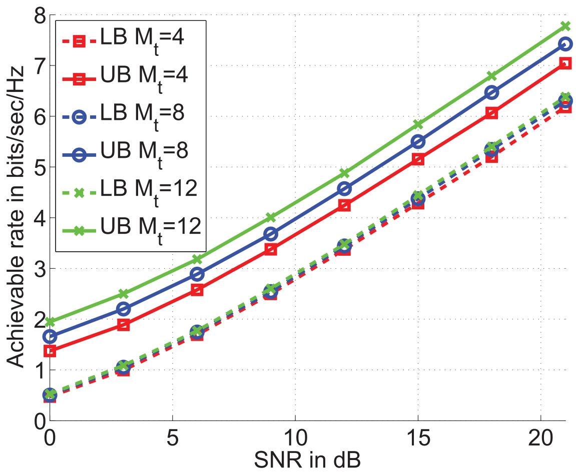

Fig. 4. Upper and lower bounds of the achievable rate of GSMM with data streams equal to the number of triggered transmit antenna ($N = M = i$, $i = 1, 2, \cdots, M_t$) in a MISO systems with $M_t = 4$, 8, and 12.

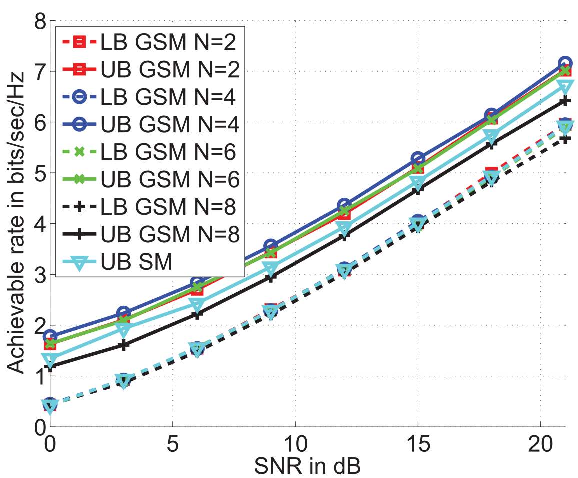

Fig. 5. Achievable rate upper and lower bounds for GSM with a single data stream ($M = 1$) in a MISO system with $M_t = 8$ and $N = 2, 4, 6$, and 8 along with the upper and lower bounds of conventional SM.

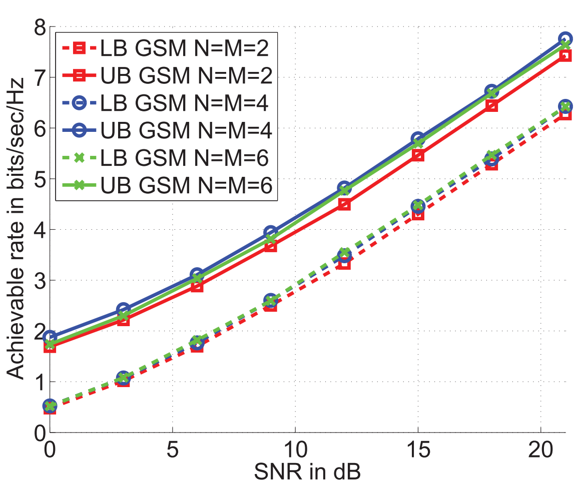

Fig. 6. Achievable rate upper and lower bounds for GSM with multiple data streams that are spatially multiplexed in a MISO system with $M_t = 8$ and $N = M = 2, 4,$ and 6.

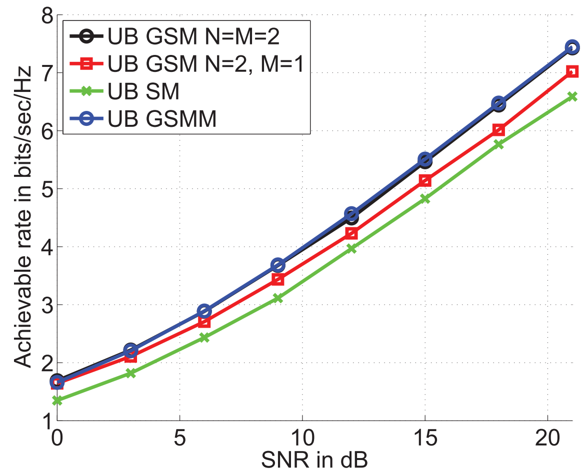

Fig. 7. Achievable rate upper bounds for SM, GSM with a single data stream ($M = 1$) and two transmit antennas activated at a time ($N = 2$), GSM with multiple spatially multiplexed data streams ($N = M = 2$), and GSMM ($N = M = i, i = 1, 2, \cdots, M_t$).

chains obviously deteriorates the energy efficiency here. From Fig. 5, it is seen that the worst option is triggering all of the transmit antennas per transmission as this makes the system lose all its spatial degrees of freedom to send the same data stream over all the transmit antennas.

The situation is different in Fig. 6 where different data symbols are multiplexed over the triggered antennas per transmission. The achievable rate seems to increase when we multiplex more data streams. It can be seen in the figure that even triggering six out of the eight antennas gives better achievable rate than triggering two out of the eight antennas as we are multiplexing more data symbols by increasing the number of triggered antennas although we have precoding sets with the same cardinality.

A comparison between all different forms of SM is presented in Fig. 7. A MISO system with eight transmit antennas is assumed and only the upper bounds of the achievable rates are shown as lower bounds are less sensitive to system changes as concluded from the previous simulations. The achievable rate of GSMM is the highest, but it has only a very slight increase (almost the same achievable rate) over the rate of GSM with two data streams multiplexed over two triggered antennas per transmission. Although slightly enhancing the performance, GSMM introduces many complications (regarding detection and receiver complexity) due to transmitting a variable number of data streams. The lowest achievable rate is for conventional SM. GSM with a single data stream and two triggered antennas per transmission offers a higher achievable rate than conventional SM, and it gets even higher when two data streams are spatially multiplexed over these two antennas as shown in the figure.

In Fig. 8, we compare the upper bound proposed in this paper with other prominent results in [27] and [28]. To be able to give a fair comparison, the same system dimension is assumed (a MISO system with $M_t = 4$) and conventional SM is utilized. The channel is assumed to be a Rayleigh fading channel which is the same assumption made in [27] and [28]. It is clear that the

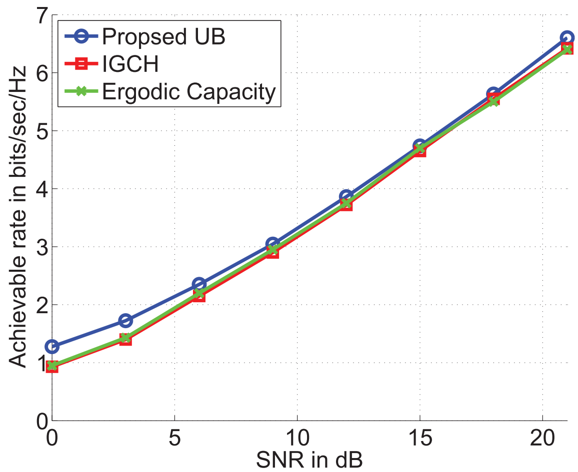

Fig. 8. Comparison of the proposed upper bound against other results in a MISO system with $M_t = 4$ in case of conventional SM ($M = N = 1$).

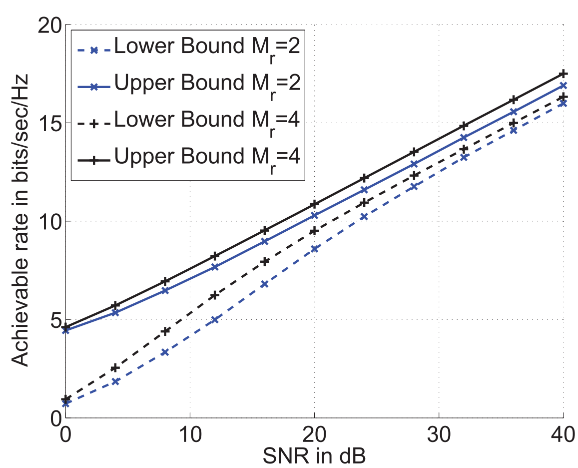

Fig. 9. Achievable rate upper and lower bounds for conventional SM ($M = N = 1$) in a MIMO system with $M_t = 8$ and $M_r = 2$, and 4.

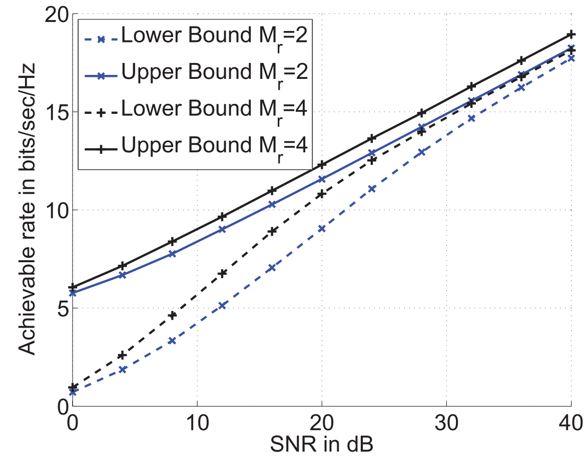

Fig. 10. Achievable rate upper and lower bounds for GSM with a single data stream ($M = 1$) and two transmit antennas activated at a time ($N = 2$) in a MIMO system with $M_t = 8$ and $M_r = 2$ and 4.

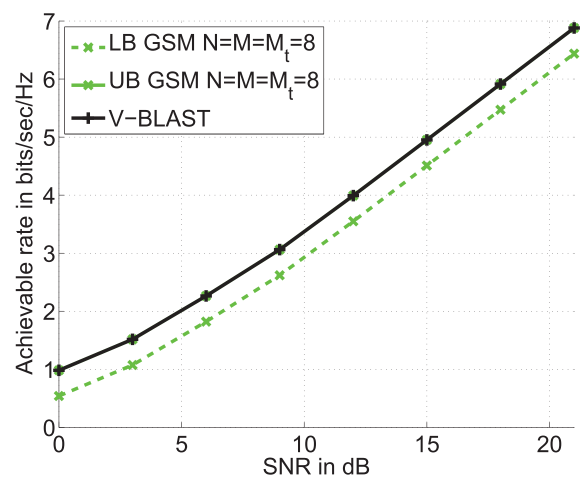

Fig. 11. Testing our bounds for the V-BLAST as a special case of GSM.

proposed upper bound is tight and the achievable rates almost overlap. Although the proposed upper bound seems to be a little bit higher at low SNR, the proposed expressions have the advantage that they can be adjusted to accommodate different SM scenarios with single, multiple, fixed, or variable data streams, while other results in [27] and [28] are restricted to MISO case only and not adjustable. Thus, the bounds are very general.

The effect of changing the number of receive antennas while fixing the number of transmit antennas ($M_t = 8$) is studied in Fig. 9 and Fig. 10 for SM and GSM with a single data stream, respectively. As it appears in the two figures, having more than one receive antenna increases the achievable rate considerably. The bounds tend to be more loose at low SNR and get tighter as SNR increases. The looseness of the bounds at low SNR is due to increasing the number of receive antennas to more than one.

Finally, we demonstrate the correctness of the proposed bounds and the tightness of the upper bound presented in this paper in Fig. 11 by testing the bounds on V-BLAST that has a well defined capacity analysis [12]. V-BLAST has an achievable rate upper bound as follows

$$C_{\text{upper bound}} = \log\det\left(\frac{\rho}{M_t}\mathbf{H}\mathbf{H}^H + \mathbf{I}_{M_r}\right).$$

Because V-BLAST can be seen as a special case of GSM, the same exact upper bound can be obtained from our proposed upper bound expression. Our proposed upper bound expression for GSM is shown in (30) to be

$$C_u^M = \log(K_M) + \sum_{i=1}^{K_M} \frac{1}{K_M} \log\det\left(\rho\mathbf{H}\mathbf{Q}_i\mathbf{H}^H + \mathbf{I}_{M_r}\right).$$

Adjusting our GSM scheme to have a number of data streams (M) that is equal to the number of transmit antennas that are all activated ($N = M_t$) and uniform triggering of all transmit antennas will correspond to the V-BLAST scenario. Applying these assumptions leads to $K_M = 1$ as the triggering of all the transmit antennas uniformly during all transmissions will lead to a precoding matrix set containing only one possible precoding matrix which is $\mathbf{F}_1 = \frac{1}{\sqrt{M_t}}\mathbf{I}_{M_t}$. Moreover, we can find the covariance matrix $\mathbf{Q}_1$ as in (8) to be

$$\mathbf{Q}_1 = \mathbf{F}_1\mathbf{F}_1^H = \frac{1}{\sqrt{M_t}}\mathbf{I}_{M_t}\frac{1}{\sqrt{M_t}}\mathbf{I}_{M_t}^H = \frac{1}{M_t}\mathbf{I}_{M_t}.$$

Substituting into the upper bound expression in (30) will give

$$C_u^M = C_u^{M_t} = \log\det\left(\frac{\rho}{M_t}\mathbf{H}\mathbf{H}^H + \mathbf{I}_{M_r}\right)$$

which is the same exact expression as the upper bound when using V-BLAST.

VII. Conclusions and Discussion

This paper studied the achievable rate expressions of a GSMM wireless system where the input symbol vector entering the precoder is assumed to have i.i.d. Gaussian entries. The distribution of the transmitted vector over the channel was shown to follow a GMM distribution. We proposed an approximate, though computationally exhausting, expression for the achievable rate of SM utilizing a precoding framework. We overcame the computational challenge by introducing a tight upper bound and a lower bound for the achievable rate that is very general and can be adjusted to accommodate different SM scenarios (SM, GSM, SSK, GSSK, and the proposed GSMM). Simulations demonstrated the effect of the dimensions of the system (number of transmit and receive antennas) on the obtained achievable rate results. We also compared our expressions with other prominent results published earlier. The tightness of the obtained upper and lower bounds and characterization of the factors that may make them loosen were also discussed.

There are still many challenges related to the practical design of GSMM receivers. As well, further improvements on the capacity expressions may be possible.

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